# An adaptive Kalman filter algorithm for estimating the altitude of vehicle

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**Abstract:** In order to better locate the altitudeof the vehicle in complex situations such as the urban environment, a high-precision and high-reliability altitude measurement algorithm is proposed. The altitudemeasurement is carried out by using three sensors: global positioning system, barometric altimeter and inertial measurement unit. The principle and model are analyzed. The state equation and measurement equation of the altitude measurement system are established according to the kinematics and sensor measurement model. The estimateof altitude is obtained by Kalman filter method. The high-information fusion method based on adaptive Kalman filter is studied and the experiment of altitude measurement system is carried out. The experimental results show that the system is designed reasonably and the altitudemeasurement has high precision.

## 1. Introduction

With the development of economy and the rapid growth of vehicles, more and more urban viaducts and parking lots are also being built up. At present, GPS/INS integrated navigation system is widely used. The inertial navigation system has high relative accuracy, good autonomy, high real-time navigation data update rate, and can continuously provide carrier motion parameters. However, its navigation positioning error increases with time, making it difficult to work independently for a long time [1]. Global positioning system features high positioning and speed measurement, high-precision speed and position information in all weather, continuous real-time, error does not accumulate over time, and the price is cheap. However, its real-time navigation update rate is poor, and when the vehicle is traveling in complex environment, the GPS satellite positioning effect is poor [2]. Combining INS and GPS can effectively utilize the advantages of INS and GPS, and complement each other. This combination can effectively reduce system errors, greatly improve navigation accuracy and reliability, and reduce navigation system cost [3]. However, in the altitude channel, the GPS positioning error will reach 10 meters or more, and the positioning error is greater in complex urban environments. The pure inertial altitude channel is unstable [4], so the pure inertia altitude channel cannot be directly used, and the external altitude information must be introduced to form a damping circuit for the altitude channel. Therefore, this paper proposes a multi-sensor vehicle altitude positioning system. Based on the original GPS/INS integrated navigation system, adding a barometer altitude measurement system and integrating altitude information of three sensors based on information fusion algorithm can obtain a low-cost, high-accuracy and reliability altitude measurement system.

## 2. The MeasuringModel

### 2.1 The Measuringmodel of GPS

$$h_{Gps} = h + \omega_1 \tag{1}$$

Where $h_{Gps}$ is the is the measurement altitude of GPS, $h$ is the true altitude of the vehicle, and $\omega_1$ is the measurement noise of GPS.

### 2.2 The measuringmodel of the barometer altimeter

Due to the influence of the gravity, the atmospheric pressure decreases with the altitude increasing. Therefore, the relationship between atmospheric pressure and altitude can be used to obtain the current altitude value through the calculation of the barometric pressure value[5]. The formula is as follows:

$$H = \frac{T_b}{\beta}\left[\left(\frac{P_H}{P_b}\right)^{-\beta R/g_n} - 1\right] + H_b \quad (2)$$

Where $H_b$ and $T_b$ are the lower limit of the gravitational potential altitude and atmospheric temperature of the corresponding atmosphere, $\beta$ is the vertical rate of change of temperature, $g_n$ is the standard free fall acceleration, $P_b$ is the lower limit of atmospheric pressure of the corresponding atmosphere, $P_H$ is atmospheric pressure of the altitude,R is the air-specific gas constant.

Then the altitude measurement equation is as follows:

$$H_{baro} = h + w_{baro} \quad (3)$$

Where $H_{baro}$ is the measuring altitude of the barometer, $h$ is the true altitude of the vehicle, $w_{baro}$ is the barometric altitude noise. Since the barometer has big noise, the differential data of the barometer altitude measured in the adjacent time interval can be used as the altitude rate to reduce the influence of the noise. That is, the mathematical model of the altitude rate of the barometer is:

$$V = \frac{{H_{baro}}^{k_2} - {H_{baro}}^{k_1}}{dt} + \omega_2 (4)$$

Where$V$ is the true altitude rate, ${H_{baro}}^{k_1}$ is the measuring altitude of the barometer at time $k_1$, ${H_{baro}}^{k_2}$ is the measuring altitude of the barometer at time $k_2$, and $dt$ is the time interval between $k_1$ and $k_2$,$\omega_2$ is the measurement noise of the barometer.

**2.3 The Measuringmodel of INS**

$$a_{ins} = a + \omega_3 \quad (5)$$

Where $a_{ins}$ is the vertical acceleration value measured by the strapdown inertial navigation, $a$ is the true vertical acceleration value, $\omega_3$ is the measured white noise, and the variance is $\sigma_g^2$.

The measured value $a_{ins}$ is calculated from the output of the strapdown inertial navigation system:

$$a = -a_x \sin\theta + a_y \cos\theta \sin\phi + a_z \cos\theta \cos\phi \quad (6)$$

Where $a_x$、$a_y$、$a_z$ are accelerations along the three axes of the vehicle, $\theta$ is the pitch angle, and $\phi$ is the roll angle.

It is assumed that the direct measurement value and the attitude angle measurement value of the acceleration obey the Gaussian distribution which is expected to be a true value, and the variances are $\sigma_\theta^2$ and $\sigma_a^2$, and are independent of each other. Then:

$$E(a) = E\big(-(a_{rx} + \tilde{a}_x)\sin(\theta_r + \tilde{\theta}) + (a_{ry} + \tilde{a}_y)\cos(\theta_r + \tilde{\theta})\sin(\phi_r + \tilde{\phi}) + (a_{rz} + \tilde{a}_z)\cos(\theta_r + \tilde{\theta})\cos(\phi_r + \tilde{\phi})\big)$$

$$= -E\cos(\tilde{\theta})\, a_{rx}\sin(\theta_r) + E\cos(\tilde{\theta})\, E\cos(\tilde{\phi}) \cdot \big(a_{ry}\cos(\theta_r)\sin\phi_r + a_{rz}\cos(\theta_r)\cos\phi_r\big)$$

$$= -e^{-\frac{\sigma_\theta^2}{2}} a_x \sin(\theta) + e^{\frac{\sigma_\theta^2}{2}} e^{-\frac{\sigma_\theta^2}{2}} (a_{ry}\cos\theta_r \sin\phi_r + a_{rz}\cos\theta_r \cos\phi_r) \quad (7)$$

It can be seen that the vertical acceleration obtained in this way is a biased quantity. If the accelerometer is changed to equation, the calculated vertical acceleration can be regarded as an unbiased measurement:

$$a = -e^{-\frac{\sigma_\theta^2}{2}} a_x \sin(\theta) + e^{\frac{\sigma_\theta^2}{2}} e^{-\frac{\sigma_\theta^2}{2}} (a_{ry}\cos\theta_r \sin\phi_r + a_{rz}\cos\theta_r \cos\phi_r) \quad (8)$$

## 3. Adaptive Kalman Filter Data Fusion Algorithm

### 3.1 Establishment of system model

Take the state variable of the system:

$$X = [hva]^T \tag{9}$$

Where $h$ is the altitude, $v$ is the altitude rate, and $a$ is the vertical acceleration.

According to the above state variables, the state equation is:

$$\dot{X}(t) = \mathrm{F(t)X(t)} + \mathrm{W(t)} = \begin{bmatrix} \dot{h} \\ \dot{v} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} h \\ v \\ a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [w_1] \tag{10}$$

The measurement equation is:

$$\begin{bmatrix} h_{gps} \\ v_{bar} \\ a_{ins} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h \\ v_{bar} \\ a \end{bmatrix} + \begin{bmatrix} \omega_1 \\ \dot{\omega}_2 \\ \omega_3 \end{bmatrix} \tag{11}$$

### 3.2 Kalman filter discretization

The state equations and the measurement equations of the Kalman filter algorithm are continuous equations, which must be linearized and discretized before they can be calculated online[6]. That is, the discrete extended Kalman filter equation for nonlinear systems needs to be designed.

The continuous Kalman filter state space equation is as follows:

$$\dot{\mathrm{X}}(\mathrm{t}) = \mathrm{F(t)X(t)} + \mathrm{W(t)} \tag{12}$$

$$Z(\mathrm{t}) = \mathrm{H(t)X(t)} + \mathrm{V(t)} \tag{13}$$

$\mathrm{X(t)}$ and $Z(\mathrm{t})$ are the system states and measured values at time $\mathrm{t}$ respectively, and $\mathrm{F(t)}$ and $\mathrm{H(t)}$ represent the system state transition matrix and measurement matrix respectively, $\mathrm{W(t)}$ and $\mathrm{V(t)}$ are the system noise and measurement noise respectively.

Discretization[7] is the process of discretizing the state transition matrix $\mathrm{F(t)}$ of the continuous system and the system noise variance matrix $\mathrm{Q(t)}$ into the state transition matrix $\Phi_{k|k-1}$ and the system noise variance matrix $\mathrm{Q}_k$.

The discretization of thestate transition matrix $\mathrm{F(t)}$ is as follows:

$$\Phi_{k|k-1} = \sum_{n=1}^{+\infty} \frac{\Delta t^n}{n!} F(\mathrm{t}_{k-1})^n = I + F(\mathrm{t}_{k-1}) \cdot \Delta t + \frac{\Delta t^2}{2!} F(\mathrm{t}_{k-1})^2 + \frac{\Delta t^3}{3!} F(\mathrm{t}_{k-1})^3 + \cdots \tag{14}$$

The discretization of the system noise variance matrix $\mathrm{Q(t)}$ is as follows:

$$\mathrm{Q}_k = \sum_{n=1}^{+\infty} \frac{\Delta t^n}{n!} \cdot M_n = M_1 \cdot \Delta t + \frac{\Delta t^2}{2!} M_2 + \frac{\Delta t^3}{3!} M_3 + \cdots \tag{15}$$

$$M_{i+1} = F(\mathrm{t}_{k-1}) M_i + (F(\mathrm{t}_{k-1}) M_i)^T i = 1,2,3 \cdots \tag{16}$$

$$M_1 = \mathrm{Q}(\mathrm{t}_{k-1}) \tag{17}$$

The basic equations for discrete Kalman filtering are as follows:

State one step prediction equation:

$$\hat{\mathrm{X}}_{k|k-1} = \Phi_{k|k-1} \hat{\mathrm{X}}_{k-1} \tag{18}$$

Where $\hat{\mathrm{X}}_{k-1}$ is the system state estimation value at time $\mathrm{t}_{k-1}$, and $\hat{\mathrm{X}}_{k|k-1}$ is the system state prediction value at time $\mathrm{t}_k$.

State estimation equation:

$$\hat{\mathrm{X}}_k = \hat{\mathrm{X}}_{k|k-1} + \mathrm{K}_k (\mathrm{Z}_k - \mathrm{H}_k \hat{\mathrm{X}}_{k|k-1}) \tag{19}$$

Filter gain equation:

$$K_k = P_{k|k-1} + H_k^T[H_k P_{k|k-1} H_k^T + R_k] \quad (20)$$

One-step prediction of mean square error:

$$P_{k|k-1} = \Phi_{k|k-1} P_{k-1} \Phi_{k|k-1}^T + \Gamma_{k|k-1} Q_k \Gamma_{k|k-1}^T (21)$$

Estimated mean square error:

$$P_k = [I - K_k H_k] P_{k|k-1} [I - K_k H_k]^T + K_k R_{k-1} K_k^T (22)$$

Kalman filter is a linear recursive algorithm. It is necessary to first determine the initial values $\widehat{X}_0$ and $P_0$ to calculate the measured value and state estimation value at the next moment. The Kalman filtering process is divided into a time update process and a measurement update process. Its recursive process is shown in Figure 1.

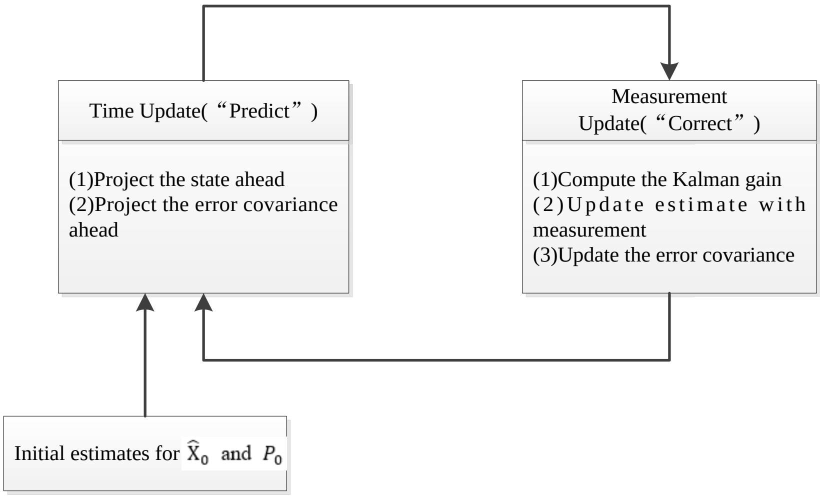


Figure 1 Recursive process of discrete Kalman filter

### 3.3 Sage-Husa adaptive Kalman filter

In the practical application of vehicle integrated navigation, due to the complexity of the actual environment, it is not a linear time-invariant system, and the statistical characteristics of system noise and measurement noise are not known a priori. In this case, ordinary Kalman filter cannot get high filtering accuracy and stability. The Sage-Husa adaptive Kalman filter[8] uses the information brought by the observations to not only adaptively adjust and update the prior information (estimation state variables) in the recursive process, but also to estimate the system noise and measurement noise in real time online.

Generally, the system noise has some stability. The measure to prevent the filter divergence is to pay attention to the status of the recent measurement value in the current filtering. Therefore, special attention should be paid to the change of the measurement noise. In order to weaken the role of the old measurement data in the estimation, the new measurement data play a major role in the estimation. The fading memory index weighting method is used to increase the weighting coefficient of the new data items. The estimation equation $R_k$ and the weighting coefficient $d_k$ of the measurement noise variance obtained by weighting coefficients are used as formulas, and the estimation equation $R_k$ and the weighting coefficient $d_k$ of the measurement noise variance are combined with the general Kalman filter equation. The simplified Sage-Husa adaptive Kalman filter algorithm can be obtained as follows:

Suppose $r_k = 0, q_k = 0$, $Q_k$ is constant, $R_k$ is unknown:

$$v_k = Z_k - H_k \widehat{X}_{k|k-1} (23)$$

$$R_k = (1 - d_k) R_{k-1} - d_k (v_k v_k^T - H_k P_{k|k-1} H_k^T) \quad (24)$$

$$d_k = (1 - b)/(1 - b^k) \quad (25)$$

$b$ is a forgetting factor and is selected by HDOP of GPS:

$$b = \begin{cases} 0.99 & HDOP \leq 2 \\ \frac{2}{HDOP} & 2 < HDOP < 5 \\ \frac{1}{HDOP} & 5 \leq HDOP < 10 \\ \frac{1}{2HDOP} & HDOP \geq 10 \end{cases} \quad (26)$$

## 4. Experiment

The data was collected in a GPS/INS vehicle integrated navigation experiment in Shanghai, and the sports time was about 50 minutes. The IMU sampling frequency is 50HZ, the GPS sampling frequency is 5HZ, the barometric altimeter sampling frequency is 50HZ, and the combined frequency is 5HZ.

The figure 2 below shows the result of the experiment.The red curve is the altitudeoutput of the altitude measurement systembased on Sage-Husa adaptive Kalman filter. The blue curve is the altitudeoutput of the GPS. And The yellow curve is the altitudeoutput of the dual antenna receiver.We use the output of the dual antenna receiver as a reference.It can be seen from the figure that the system designed in this paper effectively improves the positioning accuracy of the vehicle height measured under a single GPS, and in most cases can accurately locate, the maximum error does not exceed 2 meters.Figure 3 shows the HDOP of the GPS, which show the precision of the GPS.

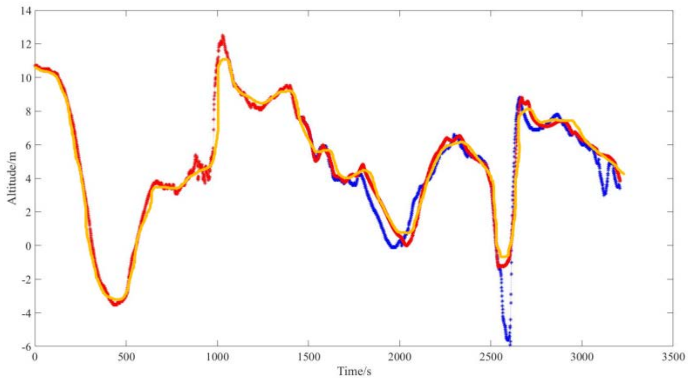


Figure 2 The result of the experiment

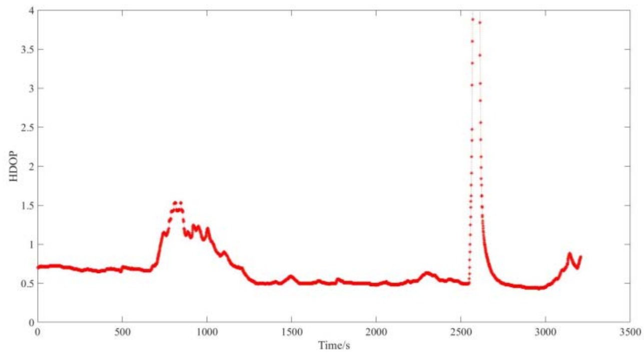


Figure 3 The HDOP of the GPS

## 5. Conclusion

This paper proposes a Sage-Husaadaptive Kalman filter algorithm. Based on the traditional GPS/INS integrated navigation, adding a barometer altitude measurement system,and thedata of the three sensors is fused and estimated by adaptive Kalman filtering, which can obtain a low-cost, high-accuracy and reliability altitude. It is verified by experiments that the accuracy of the altitude measurement system can reach less than 2 meters, which have a certain commercial value.

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